Memristor − the fourth fundamental passive electronic component and its memory interpretation

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Abstract : This paper aims to present to master students in electronics the recently discovered fourth basic passive circuit element − called memristor − and to better understand how the history memory of the already passed charge through it is taken into account in its I-V characteristic. In this attempt, a simple coupling device between 2 RC cells is investigated. Our calculations are in very good agreement with experiments on SPICE and Matlab softwares.

Keywords: Teaching, circuit elements, memristor, memory effect, charged cells, diffusion.

1 INTRODUCTION

Electronic network involves causes and effects. The causes here refer to any various voltage or current sources and the effects are part of the network using or sinking the energy source. Active elements and passive elements are the terms usually used in place of causes and effects respectively. Active circuit elements, such as voltage, current, transistors, etc, generate an electric energy or power gain to control or drive other parts of the circuit. However, passive circuit elements, such as resistor, capacitor, etc, use the available energy to function. Figure 1 shows some common electronic components and the display of their few circuitries configurations.

The mentioned basic active circuit elements are voltage and current sources in the form of independent or dependent sources. In addition, there are four fundamental circuit variables: electric current i, electric voltage v, electric charge q, and magnetic flux ϕ, where q and ϕ are defined as the time integrals of i and v respectively. These circuit variables are inter-related or linked accordingly to the basic passive circuit elements namely: resistor R, capacitor C, and inductor L, by the constitutive relationship:

$$\hat{f}(n,m) = 0,$$

where n and m could be any of i, v, q or ϕ variables. R, L and C in conjunction with the definitions of electric charge (q) and magnetic flux (ϕ) led to five known possible relationships. These three fundamental known passive circuit elements (R, L and C) in connection to other nonlinear passive circuit elements such as diodes and transistors have being the circuit components in the history of electronic system design.

However, from the symmetry argument of the circuit elements illustrated in Fig. 2, the existence of the fourth basic passive circuit element was postulated in 1971 [1], thus forming the missing sixth possible relationship. As stated in [1], for the sake of completeness there should be a fourth passive circuit element describing the relationship between magnetic flux ϕ and electric charge q - called the *memristor* (M). Memristor is the short form of memory resistor, this name is due to the fact that memristor device remembers its previous history (resistance) hence the memory effect and is analogous to resistor with memory.

The constitutive relationship of a memristor is given by $\hat{f}(\phi, q) = 0$ so that:

$$d\phi = M(q)dq \quad or \quad dq = Y(\phi)d\phi, \qquad (1)$$

for charge-controlled and flux-controlled memristor respectively. M and Y are memristance and memductance, measured in ohms (Ω) and Siemens (S) respectively. Memristance recalls the property of a memristor to remember its previous resistance state and is defined by the functional relationship between magnetic flux ϕ and electric charge q. The instantaneous memristance can be deduced from the slope of the ϕ-q locus given in the ϕ-q plane.

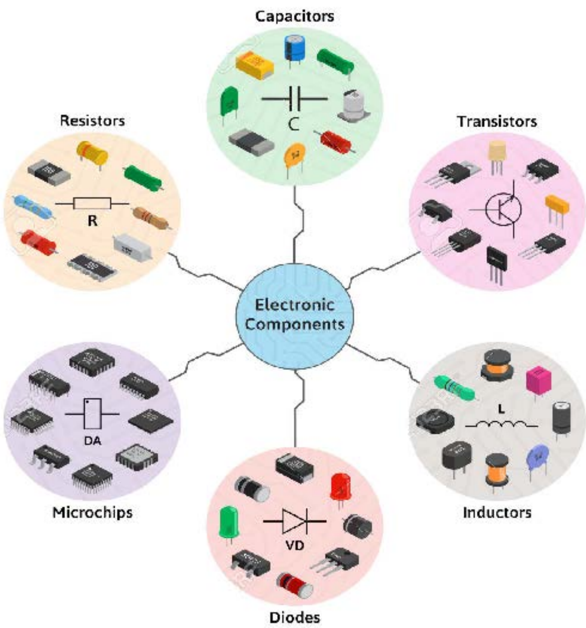

Figure 1: Display of some basic electronic components.

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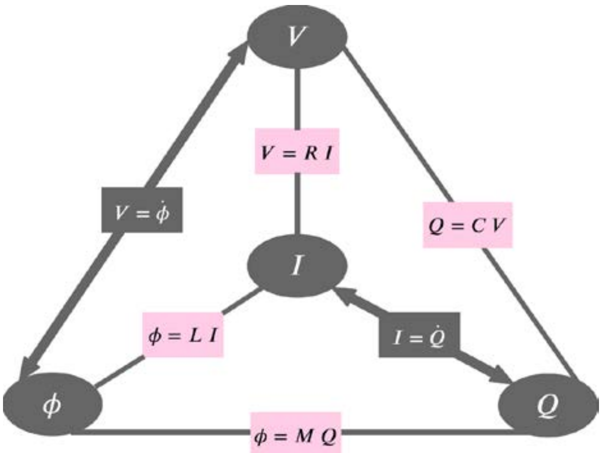

Figure 2: Symmetrical argument of the four basic passive circuit elements with respect to the four circuit variables.

Memristor is defined in [2] as: *any 2-terminal device, exhibiting a pinched hysteresis loop which always passes through the origin in the voltage-current plane when driven by any periodic input current source, or voltage source, with zero DC component. If the input is a current source, it is called a current-controlled memristor. If it is a voltage source, it is called a voltage-controlled memristor, see Fig. 3*.

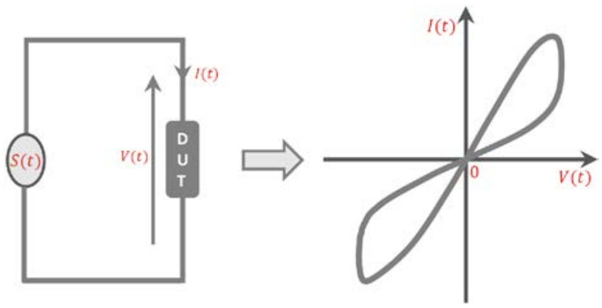

Figure 3: Memristor device under test (D.U.T) as a black box (left) and Current-voltage response of the black box (right). $S(t)$ is the periodic input source and it could be a voltage or a current source.

Circuit elements such as resistor, capacitor, inductor etc., are often characterized by their current-voltage response (*I*-*V* characteristics) in a given circuit. Memristor is not an exception, it has a peculiar current-voltage response which is a unique identifier that distinguishes it from any other known circuit element. Hence, the 3 so-called fingerprints of a memristor used to characterize memristive systems, as outlined in [3,4], are enumerated underneath.

1. The *I*-*V* response of a memristor (with positive memristance) is always a pinched hysteresis loop (Lissajous figure) when subjected to a bipolar periodic input signal.
2. The hysteresis lobe area decrease monotonically when the excitation frequency increases.
3. For a fixed input amplitude, the pinched hysteresis loop shrinks to a single-valued function as the frequency of the input supply tends to infinity.

More fingerprints of an ideal memristor are given in [5], including constitutive relationship (between flux and charge) and parameter versus state map [6]. The circuit responses of the four circuit elements is given in Appendix A, demonstrated experimentally. The memristor chip used is self-directed channel device technology fabricated by KNOWM organization [7].

2 TiO_2 MEMRISTOR

For more than three decades memristor remained a mystery until in 2008 [8] a group of researchers from HP laboratory announced the successful realization of the first memristor in device form. This recent discovery attracts the attention of many scientists, engineers and researchers to explore more feasible applications of memristor in discrete and array configurations as well as its device technology.

The HP memristor technology is made up of a thin film bilayer of Titanium-Oxide (TiO_2) of thickness *D* sandwiched between two platinum (Pt) metal contacts which serve as electrodes. One portion of TiO_2 is doped with oxygen vacancies, hence became $TiO_{2\text{-}z}$ and the other portion remained pure TiO_2. These oxygen vacancies are positively charged and thus adopt conductivity, the other undoped side has resistive property, such that the entire arrangement behaves as a semiconductor material, see Fig. 4.

Notice that in reality the charged dopants are scattered along the device width, however, its concentration in one edge is negligible compare to that of the other edge, thus causes two different resistive regions. The structural arrangement constitutes two resistances R_{on} and R_{off} connected in series. R_{on} resistance corresponds to the doped region ($TiO_{2\text{-}z}$ i.e. higher conductive region) of width (*w*) while R_{off} resistance corresponds to the undoped region (TiO_2 i.e. lower conductive region) whose width is (*D*-*w*), so that $R_{off} \gg R_{on}$ are the two resistance limits signifying OFF and ON states of the device. The boundary between doped and undoped regions (shown with two headed arrow) moves back and forth depending upon the direction of the flowing current or the polarity of the applied voltage.

Since the invention from [8], many memristor technologies emerged which are basically adhered to the principle of bipolar resistance switching between two extremes values, namely R_{on} and R_{off} that respectively correspond to the lowest and highest resistance state of the device. Another commonly used memristor technology is the self-directed channel devices [9] whose conductivity is based on the formation and dissolution of ionic bridge resulting in a low and high resistance states respectively.

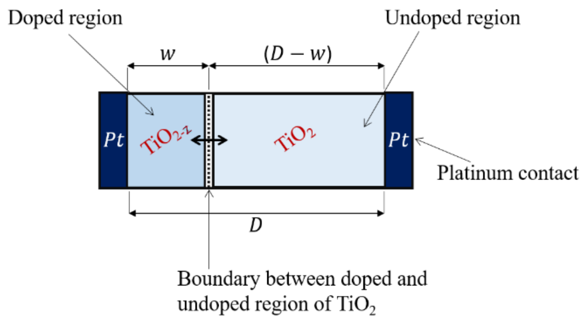

Figure 4: Structure of TiO_2 memristor. D is the width of the entire sandwiched TiO_2 layer, w is the width of doped $TiO_{2\text{-}z}$ and D-w is the width of the undoped TiO_2, so the equivalent memristance is: $M(w) = R_{on}\frac{w}{D} + R_{off}\left(1 - \frac{w}{D}\right)$.

The fabrication description of TiO_2 memristor and its equivalent mathematical model are given in Fig. 4 and equation (2) respectively, [8].

$$\begin{cases} V(t) = M(x)\, I(t), \\ \dfrac{dx}{dt} = 4\, \mu_v \dfrac{R_{on}}{D^2}\, x(1-x)\, I(t), \end{cases} \tag{2}$$

where: $x = \frac{w}{D}$ is the normalized width denoting the set of internal state variables whose value is in the range: $x \in [0,\ 1]$, $V(t)$ is the voltage across the device, $I(t)$ is the current flowing through it, w, D and μ_v are the device technology parameters. μ_v is the dopant mobility and $M(x)$ is the memristance, with $\delta R = R_{off} - R_{on}$, it can be expressed as:

$$M(x) = R_{off} - \delta R\, x, \ \ for\ \ x \in [0, 1]. \tag{3}$$

The factor $4x(1-x)$ in (2) takes into account the fact that x must remains in the range $[0, 1]$ as in Joglekar et al [10, 11] window function.

3 MEMORY EFFECT

Memristor is memory resistor, hence the memory effect of this device become a point of interest, especially in the context of application. From Fig. 4, by using

$$\frac{w}{D} \propto \frac{q}{q_d},$$

the equivalent memristance becomes [11, 12]:

$$M(q) = R_{off} - \delta R \frac{q}{q_d}, for\ q \in [0, q_d], \tag{4}$$

Where $q_d = \frac{D^2}{\mu_v R_{on}}$ is the charge scaling factor defined by the technology parameters [11]. Then, the Ohm's law $v(t) = M(q)\, i(t)$ depends not only on the current $i(t) = \frac{dq}{dt}$, but also on the initial charge q_0 having already flowed through the memristor before the initial conditions. Therefore, q_0 defines the initial memristance $M(q_0)$ of the device, so that $q_0 = \int_{-\infty}^{0} i(\tau)\, d\tau$, one can see that:

$$x(t) = \frac{q(t)}{q_d} = \frac{1}{q_d}\left[q_0 + \int_0^t i(\tau)\, d\tau\right]. \tag{5}$$

Figure 5 shows that for fixed input amplitude and frequency, the value of initial memristance affects the *I-V* characteristic of the device.

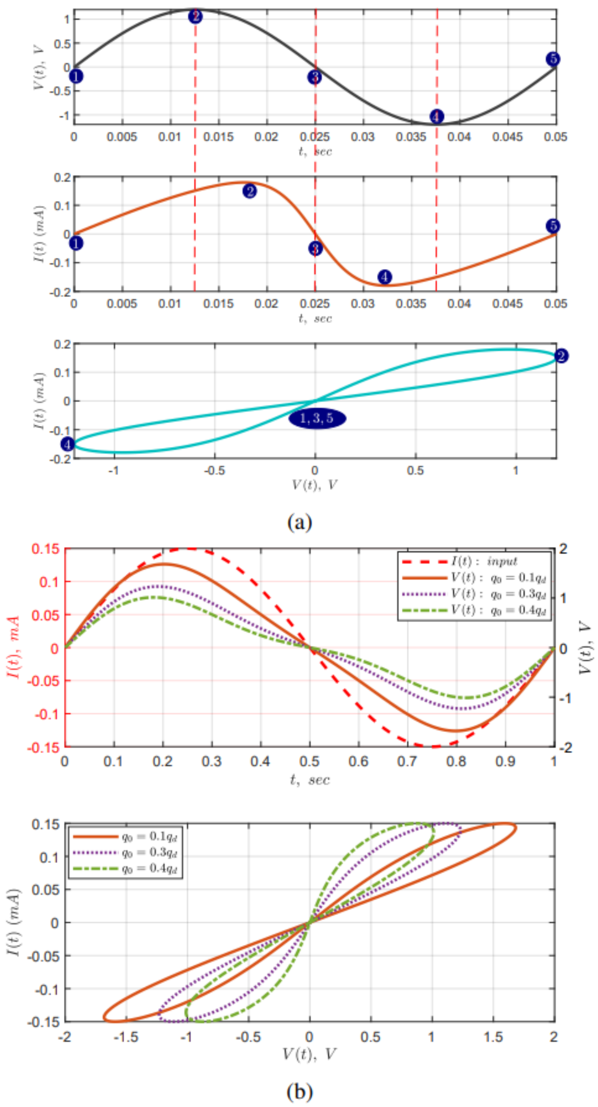

Figure 5: (a) The voltage and current waveforms in a memristor are always in phase. However, one can see that the current is not maximum even so the causative voltage is maximum (point 2), emphasizing the nonlinear nature of the device. Points 1, 3 and 5 are the evidence of pinched hysteresis loop at (0,0) i.e. $I(t) = 0$ whenever $V(t) = 0$ and vice versa. (b) Effect of initial charge q_0 on the memristor I-V characteristic, thus reflecting the memory effect of the device. $R_{off} = 16K\Omega$, $R_{on} = 100\Omega$, $I_0 = 0.15mA$, $f = 1Hz$, $\mu_v = 10f\ m^2/V.s$, $D = 10\ nm$, then $q_d = 100\mu C$ and $q_0 = 0.1q\_d$, $0.3q_d$ and $0.4q_d$.

Figure 6 shows a simple schematic with two identical RC cells coupled together by a memristor [13].

The idea is to effectively study their interaction with respect to memristor under certain initial conditions. The states of the cells are designated with subscript letters m and s, referring to master and slave cells. We choose the simplest case: $v_{m_0} > 0$ and $v_{s_0} = 0$, meaning that the current will go from the master to the slave cell. Initially, as the switches S_{w_1} and S_{w_2} are open nothing flows through the memristor. By closing the switches simultaneously at time $t = 0$ the current begins to flows, and the application of Kirchhoff's laws gives the following system of equations:

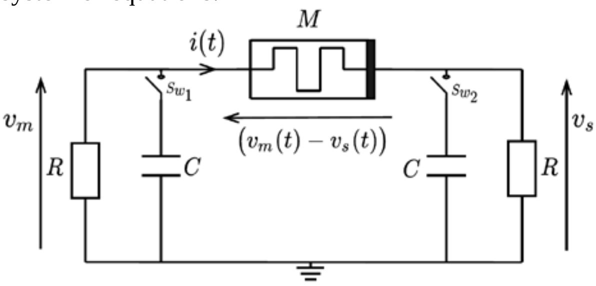

Figure 6: Two pixel cells system composed of master and a slave cells coupled together by a memristor, with the cell parameters given by the subscripts m and s respectively. M is the memristor whose model is given by equation (4).

$$i(t) = -C\frac{dv_m}{dt} - \frac{v_m}{R}, \tag{6}$$

$$i(t) = C\frac{dv_s}{dt} + \frac{v_s}{R}, \tag{7}$$

$$i(t) = \frac{dq}{dt}, \tag{8}$$

$$v_m - v_s = M(q)\frac{dq}{dt}. \tag{9}$$

From equations (6)-(8) and taken into consideration the initial conditions v_{m_0}, v_{s_0} and q_0, one can see that:

$$v_m - v_s = v_{m_0} - \frac{2R}{\tau}(q - q_0) - \frac{1}{\tau}\int_{q_0}^{q} M(q')\,dq', \tag{10}$$

where $\tau = R\,C$ is the time constant of the cells and $M(q)$ is given in (4). From (9) and (10), we get:

$$\left[\frac{\alpha + q_0 - q}{\alpha}\right]^A \left[\frac{\beta + q_0 - q}{\beta}\right]^B = e^{\frac{\delta R\,t}{2\tau\,q_d}}, \tag{11}$$

where:

$$A = \frac{1}{\alpha - \beta}\left(M(q_0) - \frac{\delta R}{q_d}\alpha\right),$$
$$B = \frac{1}{\beta - \alpha}\left(M(q_0) - \frac{\delta R}{q_d}\beta\right),$$

α and β being the characteristic roots of the 2^{nd} degree equation in $q(t)$:

$$(q^2 - q_0^2) - \frac{2q_d}{\delta R}(R_{off} + 2R)(q - q_0) + \frac{2\tau\,q_d}{\delta R}v_{m_0} = 0.$$

The time evolution of the voltage $v_m(t)$ and $v_s(t)$ is respectively given by:

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$$v_m(t) = \frac{v_{m_0}}{2}\left(e^{-\frac{t}{\tau}} + 1\right) - \frac{R_{off} + 2R}{2\tau}(q - q_0) + \frac{\delta R}{4\tau\,q_d}(q^2 - q_0^2), \tag{12}$$

$$v_s(t) = \frac{v_{m_0}}{2}\left(e^{-\frac{t}{\tau}} - 1\right) + \frac{R_{off} + 2R}{2\tau}(q - q_0) - \frac{\delta R}{4\tau\,q_d}(q^2 - q_0^2). \tag{13}$$

Therefore, choosing arbitrary values of q_0 between 0 and q_d allowed us to study the memory effect of the memristor on the evolution of $v_m(t)$ and $v_s(t)$ towards the steady state. Note that q_0 corresponds to the initial position of the boundary between doped and undoped region prior to the flow of charge $q(t)$. Consider $q_0 = 0.3\,q_d$ resulting in the initial memristance $M(q_0) = 11K\Omega$. Therefore, the circuit is simulated in SPICE by adopting the memristor SPICE model developed by Biolek et al [14], see Table 1. Additionally, equations (6)-(9) are directly simulated in Matlab numerically. Figure 7 shows the result along with comparison of the one obtained from the theoretical expressions of $v_m(t)$ and $v_s(t)$ in (12) and (13) respectively.

Table 1: SPICE model of memristor.

```
.SUBCKT memristor plus minus PARAMS: D=10N µv=10F
+ Ron=100 Roff=16K x0=0.3
** x0=q0/qd: initial state of the memristor **
* Differential equation modeling *
Gx 0 x value={ I(Emem)*µv*Ron*f(V(x))/D^2}
Cx x 0 1 IC={x0}
Raux x 0 1T
* Resistive port of the memristor *
Emem plus aux value={-I(Emem)*V(x)*(Roff-Ron)}
Roff aux minus {Roff}
* Window function for nonlinear drift modeling *
.func f(x)={4x(1-x)}
.ENDS memristor
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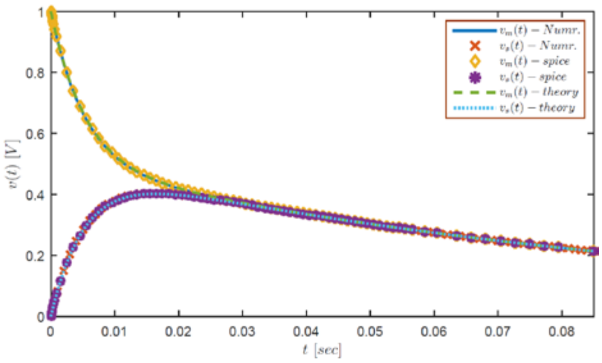

Figure 7: Comparing the evolution of $v_m(t)$ and $v_s(t)$ according to Matlab, SPICE and the theoretical calculation given by equations (12) and (13) respectively. At time $t = 0$, $v_m(t) = v_{m_0} = 1V$, $v_s(t) = 0V$, $q_0 = 0.3q_d$, $R = 100K\Omega$, $C = 1\mu F$, $R_{off} = 16K\Omega$ and $R_{on} = 100\Omega$.

To visualize the history effect of the memristor device, we considered $q_0 = 0.1q_d$, $0.3q_d$ and $0.9q_d$ as shown in Fig 8. The continuous and the corresponding dotted traces refers to $v_m(t)$ and $v_s(t)$ for a given q_0. Therefore, traces in orange, purple and green colours are for $q_0 = 0.1q_d$, $0.3q_d$ and $0.9q_d$ respectively. The time constant of the system is affected by the initial memristance: the higher is q_0, the lower is the initial memristance, hence the lower is the time constant and vice versa. This remark is drawn by looking at the respective responses of the cells under different initial conditions as depicted by Fig. 8.

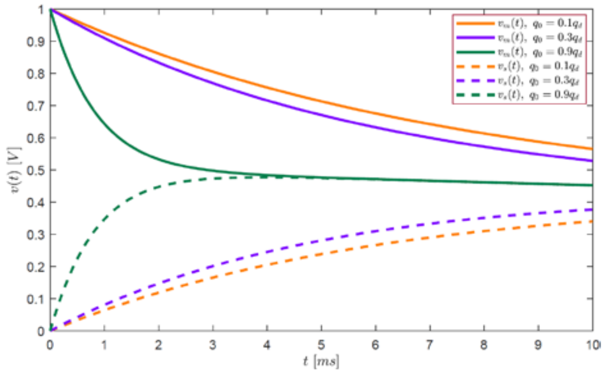

Figure 8: The evolution of $v_m(t)$ and $v_s(t)$ for $q_0 = 0.1q_d$, $0.3q_d$ and $0.9q_d$ respectively. At $t = 0$, $v(t) = v_{m_0} = 1V$, $v(t) = 0V$, $R = 100K\Omega$, $C = 1\mu F$, $R_{off} = 16K\Omega$ and $R_{on} = 100\Omega$. The result showed the effect of $M(q_0)$ on the dynamics toward the steady state.

4 EDUCATIONAL POINT OF VIEW

We have used for three years to introduce, in Electronics teaching unit for Master 1, the concept of memristor as fourth fundamental passive component, alongside with resistor, capacitor and coil. Using a periodic input voltage, Fig. 9 in appendix A shows some practical demonstrations comparing the current-voltage characteristics of the four basic circuit elements. This was achieved in front of our students with indeed part of theory, but mainly completed by practical with SPICE and MATLAB software simulations. The educational benefits are multiple:

- to train students with SPICE and MATLAB tools,
- to exercise them with a concrete case of flowing charge between cells, imitating neurons communications via synapse,
- to let them become familiar with a very promising component for their future jobs,
- and mainly to show them that Electronics is a living and fascinating science where everything is not yet discovered. While many concepts are indeed only to be developed to improve speed, miniaturization and embedded power ... more fundamental aspects could also revolutionize all jobs in electrical engineering.

5 CONCLUSION

The interpretation of the memristor as the fourth passive circuit element (alongside resistor, capacitor and inductor) is given. The work also explored the memory features of a standalone memristor and application based system. Memristor is used as a coupling device between two simple cells system which was analyzed theoretically. The theoretical result is compared with the one obtained from SPICE and Matlab simulation, showing strong agreement in the three approaches. Moreover, the memory effect of the device is investigated by considering different possibilities of the initial charge. It was shown that the memory of the device affects the dynamics of the system toward the steady state by imposing different resistance levels.

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Appendix A: Experimental demonstration of the four basic passive circuit element.

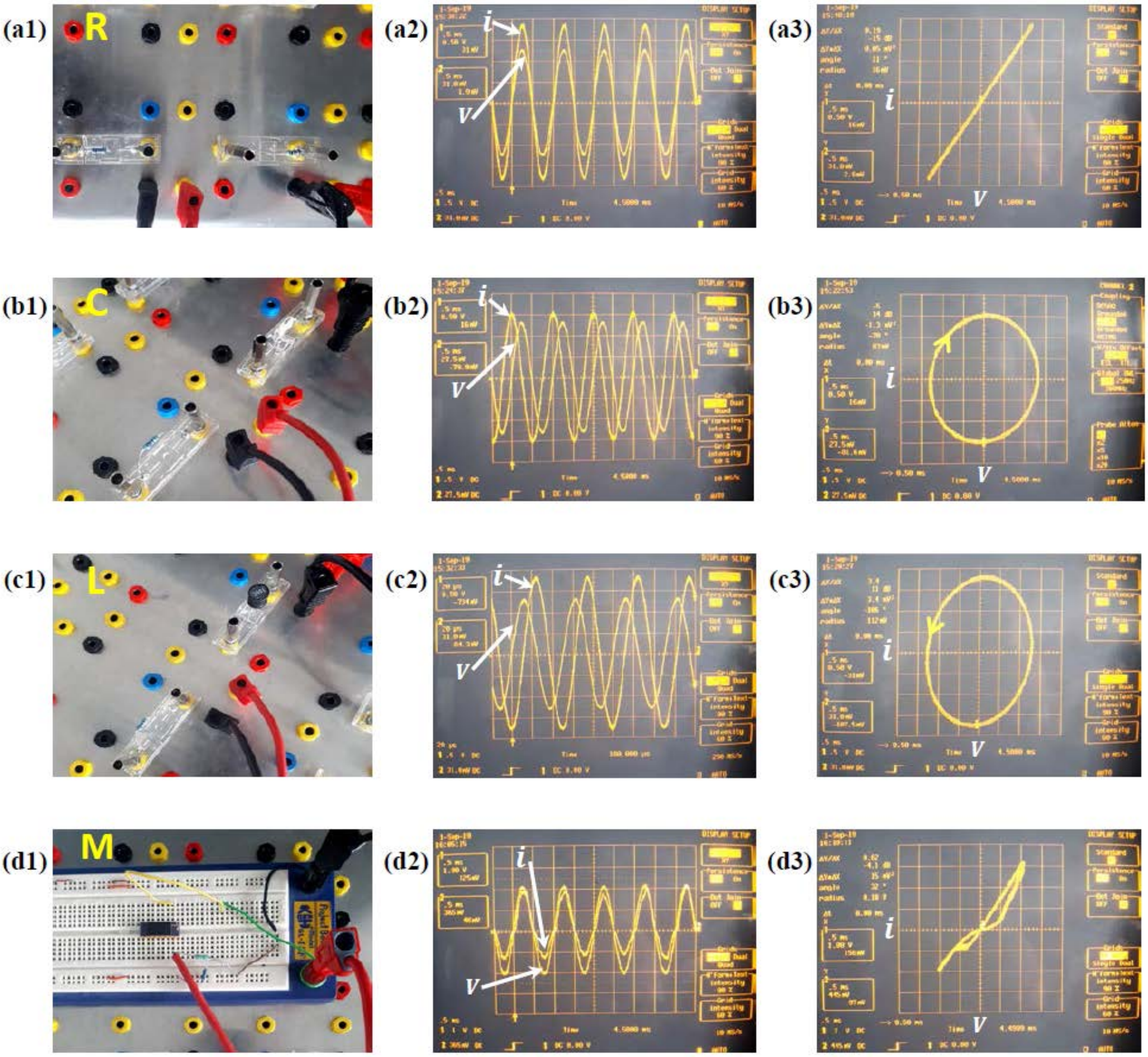

*Figure 9: Experimental results of the four fundamental passive circuit elements. **(a1-a3)** $R = 1K\Omega$, **(b1-b3)** $C = 10nF$, **(c1-c3)** $L = 10mH$ and **(d1-d3)** KNOWM memristor chip. The current through each component is measured and the corresponding I-V characteristics are given. There is no phase difference in $V(t)$ and $I(t)$ waveforms for R and M, while there is a phase difference of $\frac{\pi}{2}$ for C and L. In the capacitor C, $I(t)$ is leading the $V(t)$ by $\frac{\pi}{2}$ and in an inductor L, $V(t)$ is leading $I(t)$ by $\frac{\pi}{2}$. The I-V characteristic of R is a linear graph, for C and L it is a circle (respectively with clockwise and anticlockwise) and for M it is a pinched hysteresis loop.*

***Scales**: R: time t [0.50ms/div], current I [0.31mA/div] and voltage V [0.50V/div], C: time t [0.50ms/div], current I [0.28mA/div] and voltage V [0.50V/div], L: time t [20µs/div], current I [0.31mA/div] and voltage V [0.50V/div] and M: time t [0.50ms/div], current I [4.45µA/div] and voltage V [1.0V/div].*